**Undefined Foldable**

1. Get a half piece of construction paper.
2. Make a hot dog fold, then a burrito fold.
3. Cut along the folds in the middle of the half of the paper.

---

**Children of the Line**

- **Description**
- **Different ways to identify**
- **Examples**
Goal: Identify Points, Lines & Planes

Building Blocks of Geometry

**VOCABULARY**

- **Undefined term**: A word without a formal definition.
- **Point**: A point has no dimension. It is represented by a dot.
- **Line**: A line has one dimension. It is represented by a line with two arrowheads.
- **Plane**: A plane has two dimensions. It is represented by a shape that looks like a floor or a wall.
- **Collinear points**: Points that lie on the same line.
- **Coplanar points**: Points that lie in the same plane.
- **Defined Terms**: Terms that can be described using known words.
- **Line segment, endpoints**: Part of a line that consists of two points, called endpoints, and all the points on the line between the endpoints.
- **Ray**: The ray \( \overrightarrow{AB} \) consists of the endpoint \( A \) and all points on \( \overrightarrow{AB} \) that lie on the same side of \( A \) as \( B \).
- **Opposite rays**: If point \( C \) lies on \( \overrightarrow{AB} \) between \( A \) and \( B \), then \( \overrightarrow{CA} \) and \( \overrightarrow{CB} \) are opposite rays.
- **Intersection**: The intersection of two or more geometric figures is the set of points that the figures have in common.

**Question**: If two points are collinear are they coplanar?

**Question**: If two points are coplanar are they collinear?
Children of the Line Foldable

1. Get a half piece of construction paper.
2. Make a hot dog fold about 2/3 of the way.
3. Cut along the fold in the middle of the short half of the paper.
Goal: Segments, Rays & Distance

Line segment, endpoints Part of a line that consists of two points, called endpoints, and all the points on the line between the endpoints.

Ray The ray $\overline{AB}$ consists of the endpoint $A$ and all points on $\overline{AB}$ that lie on the same side of $A$ as $B$.

Opposite rays If point $C$ lies on $\overline{AB}$ between $A$ and $B$, then $\overline{CA}$ and $\overline{CB}$ are opposite rays.

Intersection The intersection of two or more geometric figures is the set of points that the figures have in common.

Example 2 Name segments, rays, and opposite rays

a. Give another name for $\overrightarrow{VX}$.

b. Name all rays with endpoints $W$. Which of these rays are opposite rays?

a. Another name for $\overrightarrow{VX}$ is ____.

b. The rays with endpoint $W$ are ____________________.

The opposite rays with endpoint $W$ are ____________, and ____________.

Checkpoint Use the diagram in Example 2.

2. Give another name for $\overrightarrow{YW}$.

3. Are $\overrightarrow{VX}$ and $\overrightarrow{XY}$ the same ray? Are $\overrightarrow{VW}$ and $\overrightarrow{VX}$ the same ray? Explain.

Distance The distance between two points $A$ and $B$, written as $AB$, is the absolute value of the difference of the coordinates of $A$ and $B$.

$$AB = |x_2 - x_1|$$

**POSTULATE 2 SEGMENT ADDITION POSTULATE**

If $B$ is between $A$ and $C$, then $AB + BC = AC$.

If $AB + BC = AC$, then $B$ is between $A$ and $C$. 

$$\overrightarrow{AB} \parallel \overrightarrow{BC}$$
Do Now:
(You don't need to write down the instructions.)

Step 1: Draw a number line:
Using your ruler create a number line from -8 to 8, coordinates should be 1cm apart. Label coordinate -3 with F and coordinate 5 with G.

Step 2: Fold the number line:
Fold the number line so that F lies on top of G. Unfold the paper and label the coordinate where the fold intersects the number line H. Record the coordinates in the table.

<table>
<thead>
<tr>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 3: Choose Your Own Coordinates:
Each member of the group should choose two coordinates of their number line. Label the coordinates F and G. Repeat Steps 1 & 2. Record each person's coordinates in the table.

Page 10 in Notetaking Guide

Checkpoint Complete the following exercises.

3. The endpoints of $\overline{CD}$ are $C(-8, -1)$ and $D(2, 4)$. Find the coordinates of the midpoint $M$.

4. The midpoint of $\overline{XZ}$ is $M(5, -6)$. One endpoint is $X(-3, 7)$. Find the coordinates of endpoint $Z$.

Checkpoint Complete the following exercise.

5. What is the approximate length of $\overline{GH}$, with endpoints $G(5, -1)$ and $H(-3, 6)$?
Goal: Find lengths of segments

**Distance**

The distance between two points $A$ and $B$, written as $AB$, is the absolute value of the difference of the coordinates of $A$ and $B$.

$$AB = |x_2 - x_1|$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**THE DISTANCE FORMULA**

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between $A$ and $B$ is

**POSTULATE 2 SEGMENT ADDITION POSTULATE**

If $B$ is between $A$ and $C$, then $AB + BC = AC$.

If $AB + BC = AC$, then $B$ is between $A$ and $C$.

**THE MIDPOINT FORMULA**

The coordinates of the midpoint of a segment $AB$ are the averages of the $x$-coordinates and of the $y$-coordinates of the endpoints $A$ and $B$.

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Segment bisector** A point, ray, line, line segment, or plane that intersects the segment at its midpoint.
Do Now: KWL is on your desks

<table>
<thead>
<tr>
<th>Angles and Different types of Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>What do you <strong>KNOW</strong></td>
</tr>
</tbody>
</table>

Do Now:
Write the numbers 0 to 9 on pg 42. Count the number of angles in each number.

0 1 2 3 4 5 6 7 8 9

pg 13 Notetaking Guide

**Checkpoint** Complete the following exercises.

1. Name all the angles in the diagram at the right.

2. What type of angles do the x-axis and y-axis form in a coordinate plane?
Goal: To name, classify & measure angles

**VOCABULARY**

Angle: An angle consists of two different rays with the same endpoint.

Sides of an angle: In an angle, the rays are called the sides of the angle.

Vertex of an angle: In an angle, the endpoint is the vertex of the angle.

Measure of an angle: In \( \angle AOB \), \( \overline{OA} \) and \( \overline{OB} \) can be matched one to one with real numbers from 0 to 180. The measure of \( \angle AOB \) is equal to the absolute value of the difference between the real numbers for \( \overline{OA} \) and \( \overline{OB} \).

**POSTULATE 3: PROTRACTOR POSTULATE**

Consider \( \overrightarrow{OB} \) and point \( A \) on one side of \( \overrightarrow{OB} \). The rays of the form \( \overrightarrow{OA} \) can be matched one to one with the real numbers from 0 to 180.

The measure of \( \angle AOB \) is equal to the absolute value of the difference between the real numbers for \( \overrightarrow{OA} \) and \( \overrightarrow{OB} \).

**POSTULATE 4: ANGLE ADDITION POSTULATE**

Words: If \( P \) is in the interior of \( \angle RST \), then the measure of \( \angle RST \) is equal to the sum of the measures of \( \angle RSP \) and \( \angle PST \).

Symbols: If \( P \) is in the interior of \( \angle RST \), then \( m\angle RST = m\angle RSP + m\angle PST \).

---

HW: pg 29 Textbook #1-42 (Every Other Odd)
CW Alternate: pg 29 Textbook #1-42 (Every Other Even)
Foldable: Angle pairs

<table>
<thead>
<tr>
<th>Complementary Angles</th>
<th>sum $90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplementary Angles</td>
<td>sum $180^\circ$</td>
</tr>
</tbody>
</table>

- **Complementary Angles**: Two angles whose sum is $90^\circ$.
- **Supplementary Angles**: Two angles whose sum is $180^\circ$.
- **Vertical Angles**: Two angles whose sides are opposite rays.
- **Linear Pair**: Adjacent supplementary angles.

Do Now:

**Angle Measurements in Shapes**

- What do you **KNOW**?
- What do you **WANT to know**?
- What did you **LEARN**?

Fill in the missing angles.
Goal: Special angle relationships & polygons

**VOCABULARY**

Complementary angles  Two angles whose sum is 90°

Supplementary angles  Two angles whose sum is 180°

Adjacent angles  Two angles that share a common vertex or side, but have no common interior points

Linear pair  Two adjacent angles are a linear pair if their noncommon sides are opposite rays.

Vertical angles  Two angles are vertical angles if their sides form two pairs of opposite rays.
1. Write your name on the green paper with shapes on it.
2. How many sides does each shape have?
3. Measure each angle.
4. Sum the angle measurements of each shape.
5. Write your answer on the board in the appropriate place.

Sum of the Angle Measurements

<table>
<thead>
<tr>
<th>3 sides</th>
<th>4 sides</th>
<th>5 sides</th>
<th>6 sides</th>
<th>7 sides</th>
<th>8 sides</th>
<th>9 sides</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Foldable
Goal: Learn basic polygon terms

- Classify polygons.

## VOCABULARY

**Polygon**  A polygon is a closed plane figure with the following properties: (1) It is formed by three or more line segments called sides. (2) Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

**Sides**  The sides of a polygon are the line segments that form the polygon.

**Vertex**  A vertex of a polygon is an endpoint of a side of the polygon.

**Convex**  A polygon is convex if no line that contains a side of the polygon contains a point in the interior of the polygon.

**Convex**  A polygon is convex if no line that contains a side of the polygon contains a point in the interior of the polygon.

**Concave**  A concave polygon is a polygon that is not convex.

**$n$-gon**  An $n$-gon is a polygon with $n$ sides.

**Equilateral**  A polygon is equilateral if all of its sides are congruent.

**Equiangular**  A polygon is equiangular if all of its angles in the interior are congruent.

**Regular**  A polygon is regular if all sides and all angles are congruent.

**Diagonal**  A diagonal of a polygon is a segment that joins two nonconsecutive vertices.
Do Now:

Describe the pattern in the numbers \(-7, -21, -63, -189,\ldots\) and write the next three numbers in the pattern.

Glue this onto page 48 of your Math Notebook

Finding the \(n\)th term

Copy and complete each table. Find the differences between consecutive values.

\[
\begin{array}{c|cccccccc}
  n & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
  n - 5 & -4 & -3 & -2 & & & & & \\
\end{array}
\]

\[
\begin{array}{c|cccccccc}
  n & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
  4n - 3 & 1 & 5 & 9 & & & & & \\
\end{array}
\]

\[
\begin{array}{c|cccccccc}
  n & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
  -2n + 5 & 3 & 1 & -1 & & & & & \\
\end{array}
\]

\[
\begin{array}{c|cccccccc}
  n & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
  3n - 2 & 1 & 4 & 7 & & & & & \\
\end{array}
\]

\[
\begin{array}{c|cccccccc}
  n & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
  -5n + 7 & 2 & -3 & -8 & & & & & \\
\end{array}
\]

Find the rule for the following sequence:

1. \(7, 2, -3, -8, -13, -18\)
2. \(3, 9, 15, 21, 27, 33\)
3. \(1, -2, -5, -8, -11, -14\)
4. \(-4, 4, 12, 20, 28, 36\)

Show the conjecture is false by finding a counterexample.

18. The average of any two consecutive even numbers is an even number.
19. Any four-sided polygon is a square.
20. The square of any integer is a positive integer.

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.
Goal: Describe patterns and use inductive reasoning.

VOCABULARY

Conjecture: A conjecture is an unproven statement that is based on observations.

Inductive Reasoning: Inductive reasoning is the process of finding a pattern for specific cases and then writing a conjecture for the general case.

Counterexample: A counterexample is a specific case for which the conjecture is false.

Proof: A proof is a logical argument that shows a statement is true.

Two-column proof: A two-column proof has numbered statements and corresponding reasons that show an argument in logical order.

Theorem: A theorem is a statement that can be proven.

Example 1 Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Solution

Each rectangle is divided into _______ equal regions as the figure.

Example 2 Describe the number pattern

Describe the pattern in the numbers $-1, -4, -16, -64, \ldots$. Write the next three numbers in the pattern. Notice that each number in the pattern is _______ times the previous number.

$-1, -4, -16, -64, \ldots$

The next three numbers are __________.

Example 3 Make a conjecture

Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Make a table and look for a pattern. Notice the pattern in how the number of connections _______. You can use the pattern to make a conjecture.

<table>
<thead>
<tr>
<th>Number of points</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Picture</td>
<td>.</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Number of connections</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 4 Make and test a conjecture

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

Step 1 Find a pattern using groups of small numbers.

\[
1 + 3 + 5 = 3 \cdot 3 \\
3 + 5 + 7 = ___ \cdot 3
\]
Do Now:

1. Write a conditional if-then statement.

2. Write the converse of #1.

3. Write the inverse of #1.

4. Write the contrapositive of #1.

Conditional Statements
Write a conditional if-then statement

If I go swimming, then I can't go to the park.

Converse
Inverse
Contrapositive
<table>
<thead>
<tr>
<th><strong>VOCABULARY</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional statement</strong></td>
</tr>
<tr>
<td><strong>If-then form</strong></td>
</tr>
<tr>
<td><strong>Hypothesis</strong></td>
</tr>
<tr>
<td><strong>Conclusion</strong></td>
</tr>
<tr>
<td><strong>Negation</strong></td>
</tr>
<tr>
<td><strong>Converse</strong></td>
</tr>
<tr>
<td><strong>Inverse</strong></td>
</tr>
<tr>
<td><strong>Contrapositive</strong></td>
</tr>
<tr>
<td><strong>Biconditional statement</strong></td>
</tr>
</tbody>
</table>
Do Now:

If an obtuse angle is bisected, then the two newly formed angles are
___________ and ___________.
Draw a picture to support your answer.

Triangle DGT is isosceles with TD = DG.
If the perimeter of Triangle DGT is 756 cm and GT = 240 cm, then DG = ?
Goal:

- Form logical arguments using deductive reasoning.

**VOCABULARY**

Deductive Reasoning: Using facts, definitions, accepted properties, and the laws of logic to form a logical argument.

**LAWS OF LOGIC**

- **Law of Detachment**: If the hypothesis of a true conditional statement is true, then the **conclusion** is also true.
- **Law of Syllogism**:
  - If hypothesis \( p \), then conclusion \( q \).
  - If hypothesis \( q \), then conclusion \( r \).
  - If hypothesis \( p \), then conclusion \( r \), then this statement is true.

**Example 1**: Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two angles have the same measure, then they are congruent. You know that \( m\angle A = m\angle B \).

b. Jesse goes to the gym every weekday. Today is Monday.

**Solution**

a. Because \( m\angle A = m\angle B \) satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, __________.

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "__________", and the conclusion is "__________".

“Today is Monday” satisfies the hypothesis of the conditional statement, so you can conclude that __________.

**Checkpoint**: Complete the following exercises.

1. If \( 0^\circ < m\angle A < 90^\circ \), then \( A \) is acute. The measure of \( \angle A \) is 38°. Using the Law of Detachment, what statement can you make?

2. State the law of logic that is illustrated below.

   If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show. If you do your homework, then you can watch your favorite show.

**Example 2**: Use the Law of Syllogism

If possible, use the Law of Syllogism to write the conditional statement that follows from the pair of true statements.

a. If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he will drink a glass of milk.

b. If \( x^2 > 36 \), then \( x^2 > 36 \). If \( x > 6 \), then \( x^2 > 36 \).

c. If a triangle is equilateral, then all of its sides are congruent. If a triangle is equilateral, then all angles in the interior of the triangle are congruent.

**Solution**

a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following:

**Example 3**: Use inductive and deductive reasoning

**Checkpoint**: Complete the following exercise.
Goal: Review of Algebraic Properties

ALGEBRAIC PROPERTIES OF EQUALITY
Let $a$, $b$, and $c$ be real numbers.

- **Addition Property** If $a = b$, then $a + c = b + c$.
- **Subtraction Property** If $a = b$, then $a - c = b - c$.
- **Multiplication Property** If $a = b$, then $ac = bc$.
- **Division Property** If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
- **Substitution Property** If $a = b$, then $a$ can be substituted for $b$ in any equation or expression.

DISTRIBUTIVE PROPERTY
$a(b + c) = ab + ac$, where $a$, $b$, and $c$ are real numbers.

THEOREM 2.1  CONGRUENCE OF SEGMENTS
Segment congruence is reflexive, symmetric, and transitive.

- **Reflexive** For any segment $AB$, $\overline{AB} \cong \overline{AB}$.
- **Symmetric** If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
- **Transitive** If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

THEOREM 2.2  CONGRUENCE OF ANGLES
Angle congruence is reflexive, symmetric, and transitive.

- **Reflexive** For any angle $A$, $\angle A \cong \angle A$.
- **Symmetric** If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
- **Transitive** If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.
Do Now:
Goal: Perimeter and Area

**FORMULAS FOR PERIMETER P, AREA A, AND CIRCUMFERENCE C**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>$P = \frac{4s}{s^2}$</td>
<td><img src="image" alt="Square" /></td>
</tr>
<tr>
<td></td>
<td>$A = \frac{1}{2}bh$</td>
<td><img src="image" alt="Triangle" /></td>
</tr>
<tr>
<td>Rectangle</td>
<td>$P = 2l + 2w$</td>
<td><img src="image" alt="Rectangle" /></td>
</tr>
<tr>
<td></td>
<td>$A = lw$</td>
<td><img src="image" alt="Rectangle" /></td>
</tr>
<tr>
<td>Circle</td>
<td>$C = 2\pi r$</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td></td>
<td>$A = \pi r^2$</td>
<td><img src="image" alt="Circle" /></td>
</tr>
</tbody>
</table>

Pi ($\pi$) is the ratio of a circle's circumference to its diameter.
Do Now:

part A
1.) Draw two intersecting non-perpendicular lines on the top half of your paper.
2.) Use a marker to draw a non-perpendicular line that intersects both lines from step 1 at a different intersecting point. The marker line is called the transversal.
3.) Starting from the top right angle of the transversal (marker line) label the angles a, b, c, d, e, f, g and h in a clockwise manner.
3.) Measure each angle and fill in the chart.

<table>
<thead>
<tr>
<th></th>
<th>m&lt;A</th>
<th>m&lt;B</th>
<th>m&lt;C</th>
<th>m&lt;D</th>
<th>m&lt;E</th>
<th>m&lt;F</th>
<th>m&lt;G</th>
<th>m&lt;H</th>
</tr>
</thead>
<tbody>
<tr>
<td>part A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

part B
1.) Draw two slanted parallel lines on the bottom half of your paper.
2.) Use a marker to draw a non-perpendicular line that intersects both lines from step 1 at a different intersecting point. The marker line is called the transversal.
3.) Starting from the top right angle of the transversal (marker line) label the angles a, b, c, d, e, f, g and h in a clockwise manner.
3.) Measure each angle and fill in the chart.
Goal:

- Identify angle pairs formed by three intersecting lines.

**VOCABULARY**

**Parallel lines** Two lines are parallel lines if they do not intersect and are coplanar.

**Skew lines** Two lines are skew lines if they do not intersect and are not coplanar.

**Parallel planes** Two planes that do not intersect are parallel planes.

**Transversal** A transversal is a line that intersects two or more coplanar lines at different points.

**Corresponding angles** Two angles are corresponding angles if they have corresponding positions.

**Alternate interior angles** Two angles are alternate interior angles if they lie between the two lines and on opposite sides of the transversal.

**Alternate exterior angles** Two angles are alternate exterior angles if they lie outside the two lines and on opposite sides of the transversal.

**Consecutive interior angles** Two angles are consecutive interior angles if they lie between the two lines and on the same side of the transversal.
Do Now:

Transversal Foldable: Intersecting Lines

1. \( \angle 5, \angle 7 \)
2. \( \angle 3, \angle 6 \)
3. \( \angle 1, \angle 8 \)
Goal:

**Your Notes**

**ANgles FORMED BY TRANSVERSALS**

Two angles are **corresponding** angles if they have corresponding positions. For example, $\angle 2$ and $\angle 6$ are above the lines and to the right of the transversal $t$.

Two angles are **alternate interior** angles if they lie between the two lines and on opposite sides of the transversal.

Two angles are **alternate exterior** angles if they lie outside the two lines and on opposite sides of the transversal.

Two angles are **consecutive interior** angles if they lie between the two lines and on the same side of the transversal.

**POSTULATE 15  CORRESPONDING ANGLES POSTULATE**

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are **congruent**.
Transversal Foldable: Parallel Lines

- Corresponding Angles
- Alternate Interior Angles
- Consecutive Interior Angles
- Alternate Exterior Angles
Goal: Use angles formed by parallel lines and transversals.

**POSTULATE 15**
**CORRESPONDING ANGLES POSTULATE**
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

**THEOREM 3.1**
**ALTERNATE INTERIOR ANGLES THEOREM**
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

**THEOREM 3.2**
**ALTERNATE EXTERIOR ANGLES THEOREM**
If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

**THEOREM 3.3**
**CONSECUTIVE INTERIOR ANGLES THEOREM**
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.
Do Now:

**Example 1** Apply the Corresponding Angles Converse

Find the value of $x$ that makes $m \parallel n$.

**Solution**

Lines $m$ and $n$ are parallel if the marked corresponding angles are congruent.

\[(2x + 3)^\circ = ____\] Use Postulate 16 to write an equation.

\[2x = ____\] Subtract ____ from each side.

\[x = ____\] Divide each side by ____.

The lines $m$ and $n$ are parallel when $x = ____$.

**Checkpoint** Find the value of $x$ that makes $a \parallel b$.

1. \[
\begin{array}{c}
\text{98}\degree \\
\end{array}
\]

**Checkpoint** Can you prove that lines $a$ and $b$ are parallel? Explain why or why not.

2. \[
m\angle 1 + m\angle 2 = 180\degree
\]
**Goal:**

**POSTULATE 16** CORRESPONDING ANGLES CONVERSE
If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are **parallel**.

**THEOREM 3.4** ALTERNATE INTERIOR ANGLES CONVERSE
If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are **parallel**.

**THEOREM 3.5** ALTERNATE EXTERIOR ANGLES CONVERSE
If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are **parallel**.

**THEOREM 3.6** CONSECUTIVE INTERIOR ANGLES CONVERSE
If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are **parallel**.

**THEOREM 3.7** TRANSITIVE PROPERTY OF PARALLEL LINES
If two lines are parallel to the same line, then they are **parallel** to each other.

If \( p \parallel q \) and \( q \parallel r \), then \( p \parallel r \).
Do Now:

Copy from NTG pg 71

Copy from NTG pg 73

Excerpt from the text:

Example 1  Find slopes of lines in a coordinate plane

Find the slope of line a and line c.

Slope of line a:

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{6}{4} = \frac{3}{2} \]

Slope of line c:

\[ m = \frac{6}{4} = \frac{3}{2} \]

Checkpoint Use the graph in Example 1. Find the slope of the line.

1. line b
2. line d

Example 2  Identify parallel lines

Find the slope of each line. Which lines are parallel?

Solution

Find the slope of \( k_1 \):

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

Find the slope of \( k_2 \):

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

Find the slope of \( k_3 \):

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

Compare the slopes. Because ___ and ___ have the same slope, they are ____. The slope of ___ is different, so ___ is _____ to the other lines.

Checkpoint Complete the following exercise.

3. Line c passes through \((2, -2)\) and \((5, 7)\). Line d passes through \((-3, 4)\) and \((1, -8)\). Are the two lines parallel? Explain how you know.
Goal:

- Find and compare slopes of lines.

**VOCABULARY**

**Slope** The slope of a nonvertical line is the ratio of vertical change (rise) to horizontal change (run) between any two points on the line.

**SLOPE OF LINES IN THE COORDINATE PLANE**

Negative slope: **falls** from left to right, as in line $j$.

Positive slope: **rises** from left to right, as in line $k$.

Undefined slope: **vertical**, as in line $n$.

Zero slope (slope of 0): **horizontal**, as in line $l$.

**POSTULATE 17  SLOPES OF PARALLEL LINES**

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same **slope**.

Any two **vertical** lines are parallel.

**POSTULATE 18  SLOPES OF PERPENDICULAR LINES**

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is $-1$.

Horizontal lines are **perpendicular** to vertical lines.
Do Now:

List 5 \((x,y)\) pairs of numbers whose sum is 3. (Example \((-1, 4)\)). Then plot your points on graph paper.

List 5 \((x,y)\) pairs of numbers where the difference of twice the first number and the second number is 7. (Example \((2, 3)\))
Goal: Find equations of lines.

VOCABULARY

Slope-Intercept form: The general form of a linear equation in slope-intercept form is $y = mx + b$, where $m$ is the slope and $b$ is the y-intercept.

Standard form: The general form of a linear equation in standard form is $Ax + By = C$, where $A$ and $B$ are not both zero.

Point Slope Form: Given the slope (m) of a line and a point on the line $(x_1, y_1)$.

$y - y_1 = m (x - x_1)$

Example 1: Write an equation of a line from a graph

Write an equation of the line in slope-intercept form.

Solution

Step 1: Find the slope. Choose two points on the graph of the line, $(0, 3)$ and $(2, -1)$.

$m = \frac{3 - (-1)}{0 - 2} = \frac{4}{-2} = -2$

Step 2: Find the y-intercept. The line intersects the $y$-axis at the point $(0, 3)$, so the y-intercept is $3$.

Step 3: Write the equation.

$y = mx + b$

Use slope-intercept form.

$y = -2x + 3$

Example 2: Write an equation of a parallel line

Write an equation of the line passing through the point $(1, -1)$ that is parallel to the line with the equation $y = 2x - 1$.

Solution

Step 1: Find the slope $m$. The slope of a line parallel to $y = 2x - 1$ is the same as the given line, so the slope is $2$.

Step 2: Find the y-intercept $b$ by using $m = 2$ and

$(x, y) = (1, -1)$

$y = mx + b$

$-1 = 2(1) + b$

Substitute for $x, y,$ and $m$.

$2 = b$

Solve for $b$.

Because $m = 2$ and $b = -1$, an equation of the line is $y = 2x - 1$.

Checkpoint: Complete the following exercises.

1. Write an equation of the line in the graph at the right.

2. Write an equation of the line that passes through the point $(-2, 5)$ and is parallel to the line with the equation $y = -2x + 3$. 
Do Now: What is the distance from the point to the line?

Example 4  Find the distance between two parallel lines
Railroads: The section of broad gauge railroad track at the right are drawn on a graph where units are measured in inches. What is the width of the track?

Solution
You need to find the length of a perpendicular segment from one side of the track to the other. Using Q(71, 34) and R(91, 55), the slope of each rail is

\[ m = \frac{55 - 34}{91 - 71} = \frac{21}{20} \]

The segment PQ has a slope of

\[ m = \frac{74 - 48}{29 - 15} = \frac{26}{14} = \frac{13}{7} \]

The segment PQ is perpendicular to the rail so PQ is

\[ d = \sqrt{(\frac{13}{7})^2 + (\frac{21}{20})^2} = \]  

The width of the track is ________.

Example 2  Write a proof
In the diagram at the right, \( \angle 1 = \angle 2 \). Prove that \( \angle 3 \) and \( \angle 4 \) are complementary.

Given \( \angle 1 = \angle 2 \)
Prove \( \angle 3 \) and \( \angle 4 \) are complementary.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 = \angle 2 )</td>
<td>1. Theorem 3.8</td>
</tr>
<tr>
<td>2.</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( \angle 3 ) and ( \angle 4 ) are complementary.</td>
<td>3.</td>
</tr>
</tbody>
</table>

Checkpoint  Complete the following exercises.

1. If \( c \perp d \), what do you know about the ________?

2. Using the diagram in Example 2, complete the following proof that \( \angle QPS \) and \( \angle 1 \) are right angles.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 = \angle 2 )</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( RS \perp PQ )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( \angle QPS ) and ( \angle 1 ) are right angles.</td>
<td>3.</td>
</tr>
</tbody>
</table>

Example 3  Draw conclusions
Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.

Solution
Lines \( r \) and \( s \) are both perpendicular to ________ so by Theorem 3.12, _________. Similarly, lines \( x \) and \( y \) are both perpendicular to _________. Also, lines ________ and ________ are
Goal: Find the distance between a point and a line.

VOCABULARY
Distance from a point to a line: The distance from a point to a line is the length of the perpendicular segment from the point to the line.

**THEOREM 3.8**
If two lines intersect to form a linear pair of congruent angles, then the lines are **perpendicular**.
If $\angle 1 \cong \angle 2$, then $g \perp h$.

**THEOREM 3.9**
If two lines are perpendicular, then they intersect to form four **right angles**.
If $a \perp b$, then $\angle 1, \angle 2, \angle 3, \text{ and } \angle 4$ are **right angles**.

**THEOREM 3.10**
If two sides of two adjacent acute angles are perpendicular, then the angles are **complementary**.
If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then $\angle 1$ and $\angle 2$ are **complementary**.

**THEOREM 3.11 PERPENDICULAR TRANSVERSAL THEOREM**
If a transversal is perpendicular to one of two parallel lines, then it is **perpendicular** to the other.
If $h \parallel k$ and $j \perp h$, then $j \perp k$.

**THEOREM 3.12 LINES PERPENDICULAR TO A TRANSVERSAL THEOREM**
In a plane, if two lines are perpendicular to the same line, then they are **parallel** to each other.
If $m \perp p$ and $n \perp p$, then $m \parallel n$. 
Do Now:

1. Draw a scalene and an equilateral triangle on the folder.

2. Measure the length of each side of the triangles and mark the midpoint of each side.

3. Cut out both triangles.

4. Pick a side of one of the triangles and draw a perpendicular bisector. Do it for each side in each triangle.
Goal:
Use perpendicular bisectors to solve triangle problems

**NTG pg 122**

**Perpendicular bisector**  A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a perpendicular bisector.

**Equidistant**  A point is equidistant from two figures if the point is the same distance from each figure.

**Concurrent**  When three or more lines, rays, or segments intersect in the same point, they are called concurrent lines, rays, or segments.

**Point of concurrency**  The point of intersection of concurrent lines, rays, or segments is called the point of concurrency.

**Circumcenter**  The point of concurrency of the three perpendicular bisectors of a triangle is called the circumcenter of the triangle.

---

**THEOREM 5.2: PERPENDICULAR BISECTOR THEOREM**

In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

---

**THEOREM 5.3: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM**

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

If $DA = DB$, then $D$ lies on the ⊥ bisector of $AB$. 
DO NOW:

1.) Take a piece of paper off of the overhead.
2.) Create and cut a triangle from the paper.
3.) Color each vertex of the triangle with a marker.
4.) Tear the corners about 1-2 inches from the vertex.

5.) Arrange each piece with the vertices together and the sides adjacent to form a straight line.

Compare your results with others in your group. State a conjecture of your results.

Copy the following table:

<table>
<thead>
<tr>
<th>Trial</th>
<th>m&lt;A</th>
<th>m&lt;D</th>
<th>m&lt;DCB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>8</td>
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</tr>
</tbody>
</table>
Goal: Classify triangles and find measures of their angles.

**VOCABULARY**

**Triangle** A triangle is a polygon with three sides.

**Interior angles** When the sides of a polygon are extended, the original angles are the interior angles.

**Exterior angles** When the sides of a polygon are extended, the angles that form linear pairs with the interior angles are the exterior angles.

**Corollary to a theorem** A corollary to a theorem is a statement that can be proved easily using the theorem.

**VOCABULARY**

**Leg of a right triangle** In a right triangle, a side adjacent to the right angle is called a leg.

**Hypotenuse** In a right triangle, the side opposite the right angle is called the hypotenuse.

**CLASSIFYING TRIANGLES BY SIDES**

- **Scalene Triangle** No congruent sides
- **Isosceles Triangle** At least 2 congruent sides
- **Equilateral Triangle** 3 congruent sides

**CLASSIFYING TRIANGLES BY ANGLES**

- **Acute Triangle** 3 acute angles
- **Right Triangle** 1 right angle
- **Obtuse Triangle** 1 obtuse angle
- **Equiangular Triangle** 3 congruent angles

**THEOREM 4.1: TRIANGLE SUM THEOREM**

The sum of the measures of the interior angles of a triangle is $180^\circ$.

$m\angle A + m\angle B + m\angle C = 180^\circ$

**THEOREM 4.2: EXTERIOR ANGLE THEOREM**

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

$m\angle 1 = m\angle A + m\angle B$
Do Now: Trace Triangle XYZ on tracing paper. Identify which angles in Triangle MLN correspond to the angles in XYZ.

Write congruence statements for each pair of triangles.

a. 

b. 

Proof: Complete the proof.

GIVEN: $\angle ABD \cong \angle CDB$, $\angle ADB \cong \angle CBD$, $AD = BC$, $AB = DC$

PROVE: $\triangle ABD \cong \triangle CDB$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle ABD \cong \angle CDB$, $\angle ADB \cong \angle CBD$, $AD = BC$, $AB = DC$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{BD} \equiv \overline{BD}$</td>
<td>2. ?</td>
</tr>
<tr>
<td>3. ?</td>
<td>3. Third Angles Theorem</td>
</tr>
<tr>
<td>4. $\triangle ABD \cong \triangle CDB$</td>
<td>4. ?</td>
</tr>
</tbody>
</table>
Goal: Identify congruent figures.

**VOCABULARY**

- **Congruent figures**: In two congruent figures, all the parts of one figure are congruent to the corresponding parts of the other figure.
- **Corresponding parts**: In congruent polygons, the corresponding parts are the corresponding sides and the corresponding angles.

**THEOREM 4.3: THIRD ANGLES THEOREM**

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also **congruent**.

**THEOREM 4.4: PROPERTIES OF CONGRUENT TRIANGLES**

- **Reflexive Property of Congruent Triangles**: For any triangle $ABC$, $\triangle ABC \cong \triangle ABC$.
- **Symmetric Property of Congruent Triangles**: If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.
- **Transitive Property of Congruent Triangles**: If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.
Do Now: Cut out each shape from the green paper.

Draw two triangles that have the angle measurements of 60, 80, and 40.
Goal: Use theorems and postulates to prove triangle congruence.

**POSTULATE 19: SIDE-SIDE-SIDE (SSS) CONGRUENCE POSTULATE**

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If \( \overline{AB} \cong \overline{RS} \),
\( \overline{BC} \cong \overline{ST} \), and
\( \overline{CA} \cong \overline{TR} \),
then \( \triangle ABC \cong \triangle RST \).

**POSTULATE 20: SIDE-ANGLE-SIDE (SAS) CONGRUENCE POSTULATE**

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If \( \overline{RS} \cong \overline{UV} \),
Angle \( \angle R \cong \angle U \), and
\( \overline{RT} \cong \overline{UV} \),
then \( \triangle RST \cong \triangle UVW \).

**POSTULATE 21: ANGLE-SIDE-ANGLE (ASA) CONGRUENCE POSTULATE**

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

IfAngle \( \angle A \cong \angle D \),
\( \overline{AC} \cong \overline{DF} \), and
Angle \( \angle C \cong \angle F \),
then \( \triangle ABC \cong \triangle DEF \).

**THEOREM 4.6: ANGLE-ANGLE-SIDE (AAS) CONGRUENCE THEOREM**

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

IfAngle \( \angle A \cong \angle D \),
Angle \( \angle C \cong \angle F \), and
\( \overline{BC} \cong \overline{EF} \),
then \( \triangle ABC \cong \triangle DEF \).

**THEOREM 4.5: HYPOTENUSE-LEG CONGRUENCE THEOREM**

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second triangle, then the two triangles are congruent.
Goal:

Use midsegments solve triangle problems

**Midsegment of a triangle**  A midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle.

**Coordinate proof**  A coordinate proof involves placing geometric figures in a coordinate plane.

**THEOREM 5.1: MIDSEGMENT THEOREM**

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

\[
DE \parallel AC \text{ and } DE = \frac{1}{2} AC
\]
Do Now:

Example 1  Relate side length and angle measure

Mark the largest angle, longest side, smallest angle, and shortest side of the triangle shown at the right. What do you notice?

Solution

The longest side and largest angle are ________ each other.

The shortest side and smallest angle are ________ each other.

Copy the chart below:

<table>
<thead>
<tr>
<th>Side A</th>
<th>Side B</th>
<th>Side C</th>
<th>Triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
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<tr>
<td>12</td>
<td>17</td>
<td>25</td>
<td></td>
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</tbody>
</table>

Can you make a triangle out of the side lengths? Use the toothpicks to support your answer.
Goal:

Use Inequalities in a Triangle

- Find possible side lengths of a triangle.

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Be careful not to confuse the symbol \( < \) meaning angle with the symbol \(< \) meaning is less than. Notice that the bottom edge of the angle symbol is horizontal.

THEOREM 5.10

If one side of a triangle is longer than another side, then the angle opposite the longer side is \textbf{larger} than the angle opposite the shorter side.

\[ AB > BC, \text{ so } m\angle C > m\angle A. \]

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

\[ m\angle A > m\angle C, \text{ so } BC > AB. \]

THEOREM 5.12: TRIANGLE INEQUALITY THEOREM

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

\[
\begin{align*}
AB + BC &> AC \\
AC + BC &> AB \\
AB + AC &> BC
\end{align*}
\]

Checkpoint: Complete the following exercises.

1. List the sides of \( \triangle PQR \) in order from shortest to longest.

2. Another boat makes a trip whose path has sides of 1.5 miles, 2 miles, and 2.5 miles long and angles of 90°, about 53°, and about 37°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the right.